Year 6

Progression in mathematics

The learner

At the start of the final year of Key Stage 2, children have well-developed counting skills, secure knowledge of number facts, and mental and written calculation strategies involving whole numbers and decimals. They find equivalent fractions and relate fractions to their decimal equivalents. They understand and find percentages. They use properties of 2-D shapes to complete patterns with two lines of symmetry and properties of 3-D solids to make nets. They plot coordinate points. They measure length, weight, capacity, time and angle with increasing accuracy.

During Year 6, children secure and extend their understanding of the number system to include positive and negative whole numbers and decimals with up to three places. They use their knowledge of the relationships between fractions, decimals, percentages, ratio and proportion to express quantities in a variety of ways and to solve problems. They use the properties of shapes in classification and transformations.

Children analyse and describe data using the mode, range, median and mean, and the language of chance or likelihood, predicting outcomes and solving problems. They choose effective methods to collect, process, present and interpret data, using ICT where appropriate. They evaluate the validity of their conclusions.

Systematic daily mathematics teaching, together with planned opportunities to use and apply mathematics across the curriculum, develops children's understanding and enjoyment of mathematics and their ability to use mathematics as a problem-solving tool. By posing 'What if …?' questions, children create hypotheses for themselves and others to test

Oral and mental work continues to help children to recall and derive number facts and to explain their methods and reasoning. They work collaboratively to solve problems. They take part in group and whole-class discussion about mathematics, drawing upon appropriate techniques, images, diagrams, vocabulary and resources to illustrate their ideas and conclusions. Mathematics lessons are an opportunity to analyse patterns, relationships and information, developing skills that will be useful throughout children's lives.

As the end of Key Stage 2, children can marshal a range of strategies for solving problems. They break down complex calculations into simpler steps and make informed decisions to use one or more calculation strategies. They use a calculator efficiently and accurately where appropriate.

The activities children engage in should support them in developing many of the key aspects of learning identified in *Excellence and Enjoyment: learning and teaching in the primary years*.

Using and applying mathematics

Children solve multi-step problems that involve fractions, decimals and percentages. Through discussion and application, they develop and refine their problem-solving strategies. They recognise that trying a simpler problem can point the way to the solution of a more complex problem. For example, they find the cost of 2.5 m of cable at £3 per metre, and discuss their methods. They then use these methods to find the cost of 4750 mm of cable at £3.20 per metre, and discuss the extra steps needed. Similarly, they find simple fractions and percentages of quantities such as $\frac{1}{5}$ of £400 or 10% of 50 m; they then solve more complex problems such as finding $\frac{4}{5}$ of 4.5 kg or increasing a bill of £55 by 17%.

Children consolidate their calculator skills. They decide when it is appropriate to use a calculator and what they might record to help them to keep track of their thinking. They interpret the numbers displayed and explain a remainder in the context of the problem, deciding when rounding up or down is appropriate. They understand how to use a calculator to convert a fraction to a decimal and how this helps them to order fractions.

Children work more systematically and independently and suggest, plan and develop their own line of enquiry. They recognise that representing a problem may require numbers or calculations, or organising information in a table, picture or diagram, and that the solution has to be interpreted and checked in the original context. For example, to solve a puzzle in which they are given information about who will play whom in a competition, they construct and use a table to record arrangements and eliminate possibilities. They use symbols such as letters or an empty box to represent an unknown number where appropriate.

Children make and test predictions and general statements. For example, they observe that when ninths are represented as decimals on a calculator, they have the same digit in all decimal places. They make a general statement about the decimal representations of ninety-ninths and test these with a calculator. They predict, then test, that 0.23232323 is the representation of $^{23}/_{99}$. They use words then letters as symbols to construct and use simple expressions or formulae. For example, they record that in y years there will be $12y$ months, that the nth multiple of 6 is $6n$, or that when a number a is increased by 4 it is represented as $a + 4$.

Children generate, represent and interpret sequences that involve numbers or shapes, extending sequences backwards and forwards. For example, they describe the rule for the sequence that starts with 1.2, 1.4, 2.6, 4.0, … as 'add the last two numbers', and they use the rule to generate more terms. They interpret a rule such as 'treble the last number and add 2' to find the tenth term in the sequence that starts with 3 and to find the term before 245. They interpret a strip pattern of 3 cm by 8 cm rectangles placed alongside one another in alternate positions, 3 cm then 8 cm along the base. They find the length of a strip of nine such rectangles and then the number of these rectangles that form a strip 135 cm long.

Children describe, interpret and use the patterns and relationships that they observe. For example, they recognise how a shape made of small, identical equilateral triangles can be completed to form a larger equilateral triangle. They identify the fraction the original shape was of the large equilateral triangle. They make deductions from given information to find, for example, what other angles are possible in an isosceles triangle with one angle identified as 48°. They explain their reasoning and justify their conclusions.

Counting and understanding number

Children order positive and negative whole numbers and position them on a number line. They find differences between positive and negative numbers in context; for example, they calculate the drop in temperature between 3 °C and –8 °C or the rise in temperature from –11 °C to –3 °C. They continue to count forwards and backwards across zero in whole-number or decimal steps such as 0.25. They partition and round whole numbers and decimals and use their knowledge of place value to recognise that a number such as 3.009 is less than 3.01. They position decimals with up to three places on a number line.

Children understand how to simplify fractions by cancelling, dividing the numerator and denominator by a common factor. They order a set of fractions by converting the fractions to equivalent fractions with a common denominator, for example, converting $^{2}/_{3}$, $^{3}/_{4}$, $^{5}/_{6}$ and $^{7}/_{12}$ all to twelfths. They express a larger whole number as a fraction of a smaller whole number and convert it to a mixed number. They use this to express relationships, for example, that an article costing £4, which previously cost £3, has increased by a factor of $^{4}/_{3}$ or is $1^{1}/_{3}$ times its original cost. Children also express one

quantity as a percentage of another, for example, that 70 cl is 70% of 1 litre, or that an increase of 15p on an item costing £3 represents an increase of 5%.

Children solve problems involving direct proportion by scaling quantities up or down. For example, given a recipe for four people, they change it to a recipe for one person, or to a recipe for six people.

Knowing and using number facts

Children continue to consolidate their knowledge of number facts involving all four operations and to use known facts to derive related facts. For example, they work out numbers in the 13 times-table by combining multiplication facts from the 10 and 3 times-tables. They work out products and quotients involving decimals (e.g. 0.6 × 8 and 5.6 ÷ 8) using facts from the 8 times-table. Given a fact such as 17 × 14 = 238, they work out 18 × 14 = 252 by adding a further 14. Similarly, when they multiply by a near-multiple of 10, such as 51 or 49, they multiply by the multiple of 10 and adjust by adding or subtracting the appropriate number.

Children derive quickly the square numbers to 12 × 12 and squares of multiples of 10, such as 40 × 40. They recognise that prime numbers have only two factors. They use their knowledge of multiplication and division facts to determine, say, that 47 is prime and that 51 is not prime. They find the prime factors of a two-digit number and use tests of divisibility to decide whether a number such as 342 is divisible by 2, 3, 4, 5, 6, 9 or 10. They use their knowledge of inverse operations and estimation skills to check results.

Calculating

Children add and subtract mentally whole numbers and decimals with one place. They apply their knowledge of multiplication and division facts to multiplication and division of two-digit numbers, including decimals such as 5.6 or 0.56. They use their knowledge of place value to multiply and divide whole numbers and decimals by 1000, 100 or 10, and by multiples of these, and they explain the effect. They recognise, for example, that 25 × 0.3 is equivalent to 25 × 3 ÷ 10. They use calculators to explore, for example, the effect of multiplying and dividing whole numbers by a positive number greater than 1 and a positive number less than 1.

Children use a secure, reliable method of written calculation for each operation. They recognise when one method is more efficient than another, for both whole and decimal numbers. They continue to check first if a mental method will work and then decide which method is most appropriate. They use a calculator to solve problems where several calculations are involved, using the memory to store answers to steps en route to the solution. They check results by rounding to approximate answers, explaining, for example, that 786 ÷ 38 will be about the same as 800 ÷ 40. They use divisibility tests to decide, for example, that 681 ÷ 3 will have no remainder since the sum of the digits of 681 is divisible by 3.

Children use efficient written methods to multiply and divide two- and three-digit whole numbers and decimals by one-digit whole numbers, and to multiply two- and three-digit whole numbers by two-digit numbers. They continue to approximate first and to check their answers. They are able to explain the method that they use and the steps involved.

5.65 × 9

(estimate: 6 × 9 = 54)

×	5	0.6	0.05	
9	45	5.4	0.45	50.85

Answer: $5.65 \times 9 = 50.85$

256 × 18

(estimate: 250 × 20 = 5000)

$$\begin{array}{r} 256 \\ \times \quad 18 \\ \hline 2560 \\ 2048 \\ \hline 4608 \\ {\scriptstyle 1} \end{array}$$

Answer: $256 \times 18 = 4608$

25.6 ÷ 8

(estimate: 24 ÷ 8 = 3)

$$
\begin{array}{rl}
8\overline{)25.6} & \\
-\underline{24.0} & (8 \times 3.0) \\
1.6 & \\
-\underline{1.6} & (8 \times 0.2) \\
0 &
\end{array}
$$

Answer: $25.6 \div 8 = 3.2$

45.7 ÷ 7

(estimate: 49 ÷ 7 = 7)

$$
\begin{array}{r}
6.5 \\
7\overline{)45.5} \\
\underline{42.0} \\
3.5 \\
\underline{3.5} \\
0.0
\end{array}
$$

Answer: 6.5

Children use fractions as operators to find fractions of numbers and quantities, recognising for example that $^{3}/_{10}$ of 2 metres is equivalent to three times $^{1}/_{10}$ of 200 cm, or 60 cm. They find percentages of amounts, for example, working out a 20% discount on jeans originally costing £35 a pair. They deepen their understanding of ratio. For example, they recognise that 'four black tiles to every five white tiles' is a part-to-part relationship, but that 'four out of nine tiles are black', a description of the proportion of black tiles as a fraction of the pattern, is a part-to-whole relationship.

Understanding shape

Children make and draw shapes with increasing accuracy. They apply their knowledge of the properties of 2-D shapes and 3-D solids when describing, visualising and classifying them. For example, they classify quadrilaterals using criteria such as parallel or equal sides. They investigate the properties of quadrilaterals whose diagonals intersect at right angles or whose diagonals bisect each other. Children recognise, describe and visualise solids with parallel and perpendicular edges or faces. They use this knowledge to recognise how the nets of these solids are formed and how a solid can be made from a given net.

Children use the coordinate system to identify and plot points in the first quadrant. They explore relationships between points such as (1, 3), (4, 4) and (7, 5). They plot the vertices of shapes to form, say, an isosceles triangle or a shape with reflective symmetry; for example, they identify the symmetrical trapeziums with three of their vertices at (1, 1), (1, 4) and (3, 5), or the rectangles with area 12 square units and with two of their vertices at (3, 3) and (5, 5).

Children draw shapes on grids of different types. They determine positions of shapes after a reflection, a translation or a rotation through a quarter or half turn about its centre or a vertex. They use ICT to help them to visualise, predict and confirm the position of a shape after a transformation. They recognise that the shape remains identical after one of these transformations and use this knowledge to identify congruent shapes in patterns, pictures and diagrams.

Children estimate, measure and draw angles, including reflex angles. They know that a complete turn is four right angles or 360° and they use this fact to calculate the sizes of angles around a point. They explain how they know that the angle sum of a triangle is 180°, relating this to a demonstration that the three corners torn off any paper triangle will fit together on a straight line. They calculate angles in a triangle. They measure the angles in shapes that they draw to check estimates and to test statements. For example, they explore the angles of quadrilaterals whose vertices lie on the circumference of a circle, and test the statement that the opposite angles of these quadrilaterals sum to 180°.

Measuring

Children select and use standard metric units and suitable measuring instruments to estimate, measure and check. They consolidate and extend their knowledge of the relationships between units of measure, converting between related units using decimals to two places. For example, they change 4.5 kg to 4500 g and 750 mm to 0.75 m.

Children read and interpret scales on measuring instruments where the intervals are large, or where most are unnumbered. They recognise that taking a measurement involves a comparison against an agreed standard unit and that any measurement is approximate. They explain how to calculate the size of an interval on a scale and how to estimate measurements that lie between two marked divisions. They record their results to a suitable degree of accuracy and round up or down where appropriate to the context.

Children calculate the perimeter and area of a rectangle and compound shapes that can be split into squares, half-squares or rectangles. They estimate and compare areas of irregular shapes by counting squares and part-squares.

Children extend their understanding of measurement. For example, they research the units of measure used before metric units were introduced and how some are still used (e.g. the mile, pint, gallon, stone, chain, furlong). They establish the approximate metric equivalents. They solve problems involving measures; for example, they determine how much liquid a class of 30 children drinks in a year or plan how to build a net of a container for some selected objects.

Handling data

Children develop their understanding of the language of chance and likelihood by describing situations where outcomes are equally likely, such as rolling any number from 1 to 6 on a fair dice, and situations where outcomes are not equally likely, such as picking a prime number or non-prime number from a pack of 0 to 9 digit cards. They predict and classify outcomes using vocabulary such as 'certain', 'very likely', 'unlikely' and 'impossible'. Children begin to place outcomes from observed events and experiments on a numbered probability scale to indicate the chances of occurrence, using 0 to represent 'impossible' and 1 to represent 'certain'.

Children apply their ICT skills to formulate and test hypotheses using a computer database, for example, considering what questions could be answered by analysing census data. They solve problems by collecting data, processing the data using tables or lists to draw conclusions and present their findings. They construct frequency tables with single and grouped data items and represent these as bar charts. Children use the mode, range, median and mean to represent and describe a set of data. For example, they describe the width of hand spans of children in the class using range and median, the number of TV sets in households using mode, and scores in an experiment to predict outcomes using the mean score.

Children interpret the tables and charts that they have generated using ICT, or that they have taken from a secondary source such as the Internet, a newspaper or another school subject. They interpret pie charts and use their knowledge of fractions and percentages to estimate the size of sectors. They recognise that pie charts that look similar can represent different totals and that a small sector in one pie chart may represent more items than a big sector in another.

Children extend their use of line graphs to interpret graphs where intermediate points have meaning, for example, a conversion graph showing the exchange rate between two currencies or a graph showing changes in temperature over time.

Embedding key aspects of learning

Year 6 children have the knowledge and confidence to talk in depth about mathematical concepts and to explain their solutions, decisions and reasoning. They suggest lines of enquiry based on processing information from primary and secondary sources. They predict possible and unlikely outcomes, and plan and carry out a cycle of handling data to confirm the outcomes or otherwise. They make informed decisions about how to present their information effectively, taking account of whether the data is discrete or continuous and its intended purpose and audience. Afterwards, they

assess the quality of their enquiries. They discuss how confident they are in their results, giving a rationale for their beliefs and taking into account the opinions of others.

Children use mathematics to trial different solutions. For example, they use ICT to model the effect that increasing the number of vertices has on the internal angles of a regular shape, or to find the combination of items that will leave the least change from a specified amount. They modify their steps towards a solution in the light of feedback.

Children challenge their own and others' assumptions. For example, they explain cases where dividing makes a number larger rather than smaller, or discuss whether a circle has an infinite number of vertices or none.

Children in Year 6 support each other's learning in mathematics as they revisit and consolidate previous learning from their primary years. They take responsibility for working towards achieving their own targets, evaluate the progress in their learning and offer ideas for improvement.

As they leave Key Stage 2, children have confidence in themselves as mathematicians. They know how to work independently and as a member of a group. They have developed skills of perseverance in mathematics, trying different approaches and strategies when struggling to solve a problem or to explain a concept to others. They enjoy exploring mathematical ideas and puzzles, as well as applying their mathematics in real life.