Year 4

Progression in mathematics

The learner

As they start Year 4, children have a good understanding of place value for whole numbers. They partition three-digit numbers to help them to calculate. They apply what they know about numbers, shapes and measurement to solve one- and two-step problems. They are more confident about discussing and explaining their ideas and solutions. They draw on a wider set of skills to organise and represent information and to interpret data in tables and charts. They are beginning to make connections in mathematics, to understand and use the inverse relationships between operations and to recognise proper fractions as parts of a whole. They find unit fractions of numbers and quantities.

At the beginning of the year, children can derive and recall a wide range of number facts. They are becoming more secure in using mental and informal written methods of calculation for all four operations. They add and subtract mentally one- and two-digit numbers and find sums and differences of multiples of 10. Children visualise shapes and recognise and use reflective symmetry. They use the properties of shapes to draw, make and classify the shapes. They read scales more accurately, measure, read the time and interpret time intervals. They draw and identify right angles.

As children's knowledge, skills and confidence grow during Year 4, they begin to identify more general patterns and rules. They begin to understand when and how they can use them to simplify a calculation and they reduce the steps needed to reach an answer. Children see how properties of shapes, number sequences and number sentences enable them to identify what is similar, identical or different about them. The home–school link continues to play a role. Homework helps to secure past or current learning, or prepares children for the next steps in their learning.

Systematic daily mathematics teaching develops children's understanding. Their knowledge of the number system is extended to include negative and decimal numbers. Their recall skills, and mental and written methods of calculation, are extended and refined through demonstration, discussion and regular practice. They learn multiplication tables by heart. They solve problems representing real-life situations, interpreting the solutions in their original context. Links to other subjects and to out-of-school contexts show that mathematics is a valuable problem-solving tool.

Frequent oral and mental work continues. Regular opportunities for more sustained discussion in groups and with the whole class promote thinking, explanation and reasoning skills. Children's use and application of mathematics is an opportunity to review how successfully they are acquiring the key aspects of learning in *Excellence and Enjoyment: learning and teaching in the primary years*.

Using and applying mathematics

Children solve increasingly complex word problems. Where appropriate, they use a calculator to do so. They choose the calculations that they need to do and the best way to do them. They record their methods to explain them, and interpret their answers in the context of the problem. They compare their solutions with others' and identify the best approaches. They are becoming aware that problem solving involves making decisions about the number sentences, statements, diagrams or images that they can draw upon to represent a problem, and that these are then used to solve the problem.

When they solve problems and puzzles, children make and annotate tables, diagrams or text to identify which key bits of information to use. They begin to make notes to help them to keep track of their progress. For example, they investigate the different routes through a maze on a grid and

decide how to record the pathways so that they can refer to them later. They find arrangements of coins from given information, and record their results in a table or list to keep track of the possibilities. They predict which face a 3-D shape will land on when it is dropped on the floor, and record their predictions, noting the shape and relative size of each face. They collect data by carrying out experiments, for example, rolling two dice to see if the first result is usually lower than the second. They organise and interpret their data to compare their predictions with their recorded outcomes. They analyse the data that they collect through surveys, for example, to see if more children in the school have their birthday in the summer than in the winter. They present their findings, using ICT where appropriate, to support their explanations and reasons.

Children begin to pose their own 'What if …?' questions that they and others investigate. They recognise the need to have a go, to see what works. They become more systematic, choosing ways to organise their work, and testing and checking solutions to select an appropriate strategy. For example, they use lists to record pairs of numbers with a difference of 5 in order to find the pair whose difference is 5 and whose product is 24. They refine their methods of recording as they become more confident about how to start to solve a problem and more familiar with ways of recording using diagrams, symbols and images, or ICT.

Children discuss their work, explaining patterns and rules using mathematical language and symbols. They explore the number sequences arising from a given rule such as 'double the last number and subtract 1'. They choose different starting numbers and ask and answer questions such as: 'Starting with 3, why are all the numbers odd?'; 'What if the start number is even?'; 'What are the gaps between the numbers?'; 'What if we used "add 1"?'; and 'What other rules can we investigate?'. Children use examples to test a general statement such as: 'The sum of a pair of numbers in the same column of a 100-square is always even'; 'The diagonal of a rectangle is longer than either of its sides'; or 'A hexagon can have up to four right angles'.

Counting and understanding number

Children develop their use of number vocabulary and notation. They meet negative numbers and position positive and negative numbers on a number line, for example, to order a set of temperatures. They recognise negative numbers in the display of a calculator when they use it to calculate. They write inequalities using the signs < and >, for example –3 < +5 and –5 < –3. They know that any negative number is less than any positive number.

Children generate number sequences given the start number and the whole-number step size, using a grid or number line to continue the sequence on and back. They investigate the possible size of the step if, for example, the fourth number is 11 and the eighth number is 23. They extend and use their understanding of place value to add or subtract multiples of 1, 10, 100 or 1000 to or from any four-digit whole number and to partition the numbers in different ways. They understand the importance of zero as a place holder in numbers such as 2036, partitioning it as 2000 + 30 + 6 and 1900 + 100 + 20 + 16.

Children understand the meaning of the decimal point as a separator for whole numbers and parts of a whole. They recognise the representation of tenths and hundredths and partition numbers with up to two decimal places; for example, they partition 4.75 as 4 + 0.7 + 0.05 and recognise that this is equivalent to $4 + {}^{7}\!/_{10} + {}^{5}\!/_{100}$. They write the decimal number that is equivalent, say, to four tenths and six hundredths, and position decimals on a number line. They use decimals in context. For example, they calculate in metres the amount of ribbon that is left if 65 cm is cut from a 2-metre length, and work out how £17 would be shared equally among four children.

Children extend their mathematical vocabulary to enable them to read and write fractions such as 'three fifths' and mixed numbers such as 'four and two thirds'. They count forwards and backwards in steps of one half, one quarter and one third, and position mixed numbers on a number line. They use diagrams or paper strips to identify equivalent fractions. For example, using the ITP 'Fractions'

they shade four of ten sections in a strip and two of five sections in an identical strip to show that four tenths is equivalent to two fifths. They construct a 'fraction wall' to compare fractions, for example to show that one third is more than one quarter but less than one half.

Children use practical resources to establish that one fifth of 35 kg is more than one seventh of 35 kg. They recognise the decimal equivalents of common fractions, including halves, tenths and hundredths, and use a calculator to identify decimal forms of families of fractions such as the fifths, tenths, twentieths and hundredths.

Children use the vocabulary of ratio and proportion to describe the relationship between two quantities. For example, they describe a tiling pattern as 'one red tile for every three blue tiles', or say that 'one quarter of the tiles are red'. Children share out objects in a given ratio, such as one green cube to every three yellow cubes. They keep track of the totals as they do so (4 cubes, 8 cubes, 12 cubes, and so on) to identify the multiple of 4 involved in the sequence of totals. They use their work on number sequences to predict the number of cubes of each colour in piles of 20, 40 or 400 cubes.

Knowing and using number facts

Children consolidate their knowledge of number pairs that total 100. They derive sums and differences of pairs of multiples of 10, 100 or 1000, and answer questions such as: 'What is 700 plus 500?'; 'How many more is 1400 than 900?'. Children use their knowledge of halves of two-digit multiples of 10 and of even numbers to 20 to calculate half of any even two-digit number. For example, they work out half of 58 as half of 50 plus half of 8, or as half of 40 plus half of 18. They find doubles of two-digit numbers such as 48, adding 80 and 16 to get 96. They use this knowledge to derive the doubles of 480 and 4800 and the corresponding halves of 960 and 9600.

Children learn by heart multiplication tables to 10 × 10 and derive the associated division facts. They use the vocabulary 'multiple' and 'factor' when describing relationships between numbers, for example: '24 is a multiple of 6' and '6 is a factor of 24'. They know the multiples of numbers to 10 up to the tenth multiple; for example, that the sixth multiple of 8 is 48. They make use of doubling and halving, for example, to work out 45 × 4 by doubling and doubling again.

Children use their knowledge of rounding to estimate and check calculations. For example, they reason that 224 – 88 is approximately 220 – 90 = 130; they round 87 ÷ 9 to 100 ÷ 10 or to 90 ÷ 9 to give an estimate of 10 for the answer. They round measurements to the nearest unit and amounts of money to the nearest pound. They recognise that the context can influence the need to round up or down.

Children use fractions to estimate and describe proportions, saying, for example, that: 'This container holds about half as many cubes as that one'. They use diagrams and practical resources to identify pairs of fractions that sum to 1. For example, they recognise that when a strip is divided into 10 equal sections, $^{3}/_{10}$ and $^{7}/_{10}$ of the strip make up the whole strip, or that if they turn over two fifths of five cards, the three unturned cards represent three fifths of the total of five cards.

Calculating

Children add or subtract mentally pairs of two-digit whole numbers, such as 38 + 47 and 83 – 35. Some of them may need to make jottings to record the steps. They draw on their ability to partition numbers and count on or back. For example, in the case of addition they add separately 30 + 40 and 8 + 7, and then sum 70 and 15 to get 85; or they add 40 to 38 to make 78, then add 7 to get 85. In the case of subtraction they count on, adding 5 to 35 and 43 to 40, then adding 5 and 43 to get the difference of 48. Alternatively, they subtract 30 from 83 to get 53, and a further 5 to get 48. They discuss their methods and look for methods that they can do most easily in their heads with little or

no recording. Children use these mental methods to find the missing numbers in number sentences such as □ + 54 = 86, or 94 – n = 52.

Children apply their mental calculation skills to add and subtract multiples of 10 and 1000. For example, they work out what to add to 370 to make 1000, or 910 minus 740. They find the difference between two near numbers such as 7003 and 6988 by bridging across 7000 and adding 3 and 12 to get the answer 15. Where necessary, they continue to use jottings such as a number line to support mental calculations and to record methods that they explain to other children.

Children recognise the need for conventions and rules when carrying out calculations involving more than one addition or subtraction. For example, they recognise that the answer to the calculation 9 – 5 – 3 is 1 and that the calculation is carried out from left to right – otherwise a different answer is obtained (if 5 – 3 is carried out first the answer is 7). Children test the effect that changing the order in which they carry out the steps in the calculation has on the answer. For example, they use a calculator to work out groups of calculations such as 24 + 29 – 47, 29 + 24 – 47, 24 – 47 + 29 and 29 – 47 + 24, and explain why the answers are the same. They recognise that addition can be done in any order and a calculation of the type A – B + C can be rewritten as A + C – B, and either A + C or C – B can be done first. They apply the rule to calculations such as 12 – 17 + 19 that they carry out mentally, rearranging this to 12 + 19 – 17 to avoid negative numbers.

Children build on their understanding of place value and partitioning to refine and use written methods of recording for the addition and subtraction of two- and three-digit numbers. They always check first to see if they can do the calculations in their heads. For example, they recognise that they can work out 50 + 76 and 60 – 28 in their heads, but that to answer 341 + 176 or 213 – 76 they need to record steps to help them. They begin to understand how the methods that they use relate to each other and, for particular calculations, why some methods are more efficient than others.

574 – 186

$$\begin{array}{rl} 574 & \\ -186 & \\ \quad 14 & \rightarrow 200 \\ +374 & \rightarrow 574 \\ \quad 388 & \end{array}$$

Children use their skills of partitioning to support the expanded method of calculation:

237 + 185

$$\begin{array}{rrrrrrr} 200 & + & 30 & + & 7 & & \\ +100 & + & 80 & + & 5 & & \\ \hline 300 & + & 110 & + & 12 & = & 422 \end{array}$$

723 – 458

$$\begin{array}{rrrrrrr} 600 & + & 110 & + & 13 & & \\ -400 & + & 50 & + & 8 & & \\ \hline 200 & + & 60 & + & 5 & = & 265 \end{array}$$

As children become more confident in using expanded methods, recording as much detail becomes less essential. Some children begin to use a more compact method of recording for addition and subtraction.

Children explain, for those numbers that give a whole-number answer, the effect of multiplying and dividing by 10 and 100. They recognise that multiplying by 10 and then by 10 again is equivalent to multiplying by 100. They use the language of scaling up and down, recognising, for example, that a centimetre is 100 times smaller than a metre and that distances on a map are scaled up by multiplying by a factor such as a multiple of 100. They derive answers to calculations such as 30 × 5, working this out as 3 × 5 × 10 = 15 × 10 = 150. They add fives to find 31 × 5, 32 × 5, and so on, and subtract fives to find 29 × 5, 28 × 5, and so on. They find £2400 ÷ 20 by dividing by 10 and then halving. They also recognise that dividing, say, 2400 by 10 gives 240 and dividing by 10 again gives 24, and that this is the same as dividing 2400 by 100.

Children develop written methods for short multiplication and division calculations such as 74 × 4 or 87 ÷ 6. They partition the two-digit number and use a grid method for multiplying:

37 × 4

×	30	7	
4	120	28	148

Answer: $37 \times 4 = 148$

Children divide by subtracting multiples of the divisor. They check to see if the divisor multiplied by a multiple of 10 can be used and look for the largest possible case. For example, when they divide 64 by 4, children recognise that the answer must lie between 40 ÷ 4 = 10 and 80 ÷ 4 = 20. They use this approximation to do a calculation either by partitioning the two-digit number or by repeated subtractions, starting with the largest multiple of 10:

64 ÷ 4 = (40 + 24) ÷ 4
= (40 ÷ 4) + (24 ÷ 4)
= 10 + 6 = 16

64
−40 (4×10)
24
−24 (4×6)
0

Answer: 16

Children recognise that a remainder represents what is left over after a division and that it is always smaller than the divisor. They give a remainder as a whole number and make sensible decisions about rounding up or down after division according to the context of the problem.

96 ÷ 7 = (70 + 26) ÷ 7
= (70 ÷ 7) + (26 ÷ 7)
= 10 + 3 R 5 = 13 R 5

96
−70 (7×10)
26
−21 (7×3)
5

Answer: 13 R 5

Children apply their knowledge of multiplication and division to one- and two-step calculations involving money, measures and time. For example, they calculate the change from £5 when they buy five oranges at 35p each, giving the answer in pounds and pence. They work out the number of 250 ml bowls of soup that can be filled from a 2-litre pan. They find the number of seconds in six and a half minutes. They use calculators where appropriate, understanding and recording the steps involved, and checking and interpreting the number displayed in the context of the question. They understand that 3.5 on a calculator means £3.50 in the context of money.

Children use their knowledge of multiplication and division to find fractions of numbers and quantities. For example, they find $\frac{1}{5}$ of £40 by dividing 40 by 5. They use practical resources or diagrams to find proper fractions. For example, they use squared paper or an interactive whiteboard to work out $\frac{5}{8}$ of a 12-by-4 rectangle, first working out and colouring in the squares that represent one eighth of the rectangle and then finding and colouring four more such groups.

Understanding shape

Children describe and classify an increasing range of 2-D and 3-D shapes. They recognise and draw regular and irregular polygons using templates, grids or ICT, such as the ITPs 'Fixpoints' or 'Polygon'. They identify lines of symmetry in the shapes and other properties such as equal sides and equal angles. They visualise familiar 3-D shapes from 2-D drawings and make and identify the nets of common solids such as the cube, cuboids and simple prisms.

Children know that angles are measured in degrees and that a quarter turn or one right angle is 90 degrees. They use this to work out that one whole turn is four right angles or 360 degrees. They calculate that half a right angle is 45 degrees and one third of a right angle is 30 degrees. They make and use templates of 90-, 45- and 30-degree angles to build other angles and to compare against angles in polygons. They compare and order angles less than 180 degrees and begin to estimate their size.

Children talk about horizontal and vertical lines and identify horizontal and vertical faces or edges of cuboids or prisms placed on a table. They use the eight compass points to describe direction and

identify squares on a grid using a code such as A3. They interpret movement about a grid; for example: 'If you move south east from square C6, which squares will you pass through?'

Measuring

Children choose and use standard units to measure. They record estimated and measured lengths, weights and capacities using decimal notation where appropriate, e.g. 1.35 metres. They understand and use the prefixes kilo-, centi- and milli- to write, say, 4125 grams as 4 kilograms and 125 grams. They relate standard measures to real-life contexts, as in: 'My hand is about 6 cm wide' or: 'I drink about a quarter of a litre of water at lunch time'.

Children interpret partly numbered scales. They work out the value of each division and count between the numbered divisions to confirm their interpretation. They record their readings using whole numbers or decimals, for example as 5.6 grams or 1.45 litres.

Children read times to the nearest minute from analogue and digital clocks. They use 12-hour clock notation and am and pm, recognising that 2:45 pm is the same as a quarter to three in the afternoon. They choose the unit of time to estimate or measure time intervals, such as the time they take to walk across a room or the time to travel by plane to another country. They interpret timetables and use them to calculate, say, how long a journey should take, using a time line as support.

Children understand 'perimeter' and 'area' and find the perimeter and area of a rectangle by measuring the sides and by counting squares. They create shapes on a square grid and find the area, giving the answer in square units. For example, using a centimetre grid, they draw different shapes with a perimeter of 24 cm. They look for the shape with the biggest area and decide if any of their shapes has an area of 13 square centimetres.

Handling data

Children collect the information they need to answer a question. They organise the data and present and analyse it to look for patterns, trends or unusual outcomes. They extract data from tables, graphs and charts in order to highlight and illustrate points. For example, from temperature data collected in science or population data in geography, children describe the trend over time or look out for dips or peaks that signal some unusual event. They use a range of graphs and diagrams to present and explain their work to other children.

Children extend their skills by representing and interpreting data in pictograms where one symbol represents 2, 5, 10 or 20 units. They use bar charts with intervals labelled in 2s, 5s, 10s or 20s. They use ICT to compare the way that different scales or different diagrams can change the impact of the representations. They understand that the more items a symbol in their pictogram represents, the fewer symbols there will be. They recognise that as the size of the intervals on a bar chart changes so does the relative size of the bars, and that when an interval represents a larger number, differences in readings are less obvious.

Children continue to use two-way Venn and Carroll diagrams. They use these diagrams to display information about shapes and numbers, drawing on their knowledge and understanding of the properties.

Embedding key aspects of learning

Year 4 children are beginning to acquire the vocabulary and knowledge that enables them to talk in more depth about numbers and shapes and to explain their solutions, decisions and reasoning. They suggest a line of enquiry and investigate statements involving numbers that they can test with

examples. Their reasoning and problem-solving skills develop as they meet and try to solve logical mathematical puzzles and two-step word problems.

This greater independence is supported by collaborative activity in which children work with others. They develop their social skills as they listen and respond to other children's ideas. 'What if …?' questions help to develop their creative thinking. Their communication skills are extended as they learn new mathematical vocabulary and use this to interpret solutions in the context of the problem.

The introduction of written methods that build on earlier practical, mental and visual work, and consideration of why some calculation methods are more efficient than others, helps children to develop their evaluation skills.

Children extend their information processing skills. They learn more ways to organise and present information, drawing on ICT to help them. They identify patterns and pose and test conjectures. The analysis of information in written, pictorial and symbolic form takes on a more significant role, and challenges their thinking and reasoning.

During Year 4, children develop a greater awareness of how they learn. They are increasingly able to observe their own progress and to identify their own learning goals.